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# EFFECTS OF LINEAR VELOCITIES CAUSED BY GUN PLATFORM MOTION ON GUN FIRING ACCURACY

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WEAPONS SYSTEMS DEPARTMENT

**SEPTEMBER 1984** 

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Linear velocities imparted to a projectile fired from a movin of hitting (Ph) a target. These velocities are not usually adjuste 2000m, it is shown for a typical platform motion that the Ph 2.29m <sup>2</sup> target is lowered from .453 to .348. A method of method	d by the gun stabilization system. At for a 25mm gun shooting at a 2.29 x

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pensated by the fire control system is discussed.

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#### **FOREWORD**

This report explains how velocities and projectiles from guns mounted on a moving vehicle are affected by linear velocities of the vehicle. A method of modeling aiming errors caused by such linear velocities, if uncompensated by the fire control systems, is explained.

This work was done in support of accuracy model efforts on the proposed LVT(X) amphibian vehicle and the Joint Munitions Effectiveness Manual Surface-to-Surface Burst Fire Accuracy Program.

This report was reviewed and approved by R. G. Hinkle; G. E. Hornbaker, Head, Systems Accuracy Branch; and D. S. Malyevac, Head, Systems Analysis Division.

Approved by:

R. J. ARTHUR, Deputy Head Weapons Systems Department

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#### INTRODUCTION

Along with the mean velocity that takes a land or water vehicle from one place to another, there are six types of oscillatory motions, which are rotational—roll, pitch, and yaw; and translational—surge, sway, and heave. These six types of motion, as well as the mean vehicle velocity, cause linear velocities that add to the muzzle velocities of projectiles fired from guns located on a moving vehicle.

The stabilization system of a gun can compensate for part or all of the angular deviations of the aimpoint caused by roll, pitch, and yaw. However, the linear velocities imparted by these rotations as well as the translations are not accounted for by the stable element. Moreover, there are some fire control systems in use that do not account for the same, and so in these systems, the unaccounted velocities affect gun fire accuracy. This report provides a method to model the errors caused by these linear velocities, if uncompensated by the fire control. Given some assumptions, the formulas derived are exact with absolutely no approximations, and they can be utilized by writing a computer program.

This work was necessary since the operational requirements of the LVT(X) amphibian vehicle included the firing of its weapon from water near the shore. Also, the Joint Munitions Effectiveness Manual (JMEM) Surface-to-Surface Delivery Accuracy Working Group needed a method to model gun fire errors due to firing from a moving platform.

#### **METHODOLOGY**

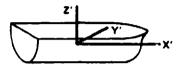
In order to derive the model, equations that calculate the linear velocity in the (North, West, up) frame due to rotational motion are explained. Transformations are performed on all other motions of the vehicle and muzzle velocities to express them in the above frame. The velocities are then added to the muzzle velocity, and the azimuth and elevation formulas for the new velocity are derived from the sum. The errors are calculated by comparing the new azimuth and elevation with the initial ones from muzzle velocity. A computer program using the methodology of this report is given in Appendix A.

#### LINEAR VELOCITY DUE TO ROTATIONAL MOTION

It is assumed that the gun barrel is pivoted on the trunnion. The stabilization system can adjust the gun barrel to compensate for angular deviations caused by vehicle rotational oscillations, but it cannot adjust for the linear velocities. Since the pivot of the gun barrel is assumed to be at the trunnion, the linear velocity imparted on the projectile from vehicle motion is equal to the linear velocity on the trunnion. This is because the gun barrel and

muzzle do not rotate with the vehicle (due to the stable element) while the trunnion does. A small additional velocity imparted while the projectile travels down the gun barrel will be discussed in Appendix B.

Assuming that the centers of rotation of roll, pitch, and yaw coincide, define X' pointing towards the fore, Z' pointing up, and Y' pointing in the direction that forms a right-handed system. Also, X' is in the centerline of the vehicle, and the point (0,0,0) is in the center of rotation.



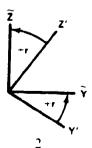
An equation will now be derived for the linear velocity with respect to a (North, West, up) frame imparted by the rotational motion.

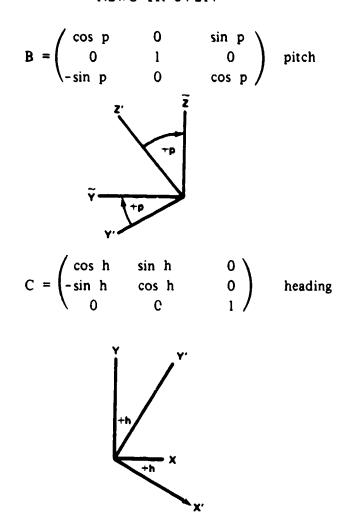
Let r = roll, p = pitch, h = heading; assuming the angle of the mean vehicle velocity vector is constant,

$$\frac{dh}{dt} = \left(\frac{d \ YAW}{dt}\right) \tag{1}$$

Also, let A = roll, B = pitch, C = heading coordinate rotations. The positive direction of roll and heading are opposite of the conventions in a right-handed system. The  $\sim$ s in the following frames show that the axis is not in the (North West, up) frame.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos r & \sin r \\ 0 & -\sin r & \cos r \end{pmatrix} \text{ roll}$$





Multiply CBA by the position vector (R') of the trunnion in the primed (platform) frame gives its position vector in the (X, Y, Z) = (North, West, up) = (N,W,U) frame, which is assumed to be fixed (Equation (2)). Hence,

$$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = C B A \begin{pmatrix} R'_x \\ R'_y \\ R'_z \end{pmatrix} i.e., R = CBAR'$$
(2)

where  $R'_x$  is the roll,  $R'_y$  is the pitch, and  $R'_z$  is the heading and yaw axes.

The linear velocity with respect to (X,Y,Z) is simply the derivative of R = R(t) as shown in Equation (3).

$$\begin{pmatrix} V_{x} \\ V_{y} \\ V_{z} \end{pmatrix} = \begin{pmatrix} \frac{dR_{x}}{dt} \\ \frac{dR_{y}}{dt} \\ \frac{dR_{z}}{dt} \end{pmatrix} = \frac{d}{dt} \begin{bmatrix} C & B & A & \begin{pmatrix} R'_{x} \\ R'_{y} \\ R'_{z} \end{bmatrix} \end{bmatrix}$$
(3)

In order to calculate the derivative on the right-hand side of Equation (3), the following definition must be made by Equation (4). Given matrix

$$M = (M_{ij}) \tag{4}$$

Equation (5) is the definition of the derivative of a matrix, which is the derivative of every element in the matrix.

$$\frac{dM}{dt} = \left(\frac{dM_{ij}}{dt}\right) \tag{5}$$

Equation (6) is a mathematical theorem. Given matrices  $A_1$ ,  $A_2$ , ...  $A_n$  and vector R

$$\frac{d}{dt} \prod_{i=1}^{n} A_{i} R = \sum_{i=1}^{n} (A_{i} \dots A_{n}R) + \prod_{i=1}^{n} A_{i} \frac{dR}{dt}$$
(6)

The proof is given in Appendix C.

From Equations (4) and (5),  $(V_x, V_y, V_z)$  can be calculated as follows.

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \frac{dC}{dt} BAR' + C \frac{dB}{dt} AR' + CB \frac{dA}{dt} R' + C B A \frac{dR'}{dt}$$
(7)

Note that the last term on the right is zero, since R' is assumed fixed on the vehicle coordinate system. Hence,

$$V_{rot} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \frac{dC}{dt}BAR' + C\frac{dB}{dt}AR' + CB\frac{dA}{dt}R'$$
 (8)

where

 $\mathbf{V}_{\mathtt{rot}}$  represents the linear velocity imparted by rotational motion.

The matrix derivatives shown are obtained by differentiating each element of the original matrices.

$$\frac{dA}{dt} = \frac{d}{dt} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos r & \sin r \\ 0 & -\sin r & \cos r \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sin r & \frac{dr}{dt} & \cos r \frac{dr}{dt} \\ 0 & -\cos r & \frac{dr}{dt} & -\sin r & \frac{dr}{dt} \end{pmatrix}$$

$$\frac{dB}{dt} = \begin{pmatrix} -\sin p \frac{dp}{dt} & 0 & \cos p \frac{dp}{dt} \\ 0 & 0 & 0 \\ -\cos p \frac{dp}{dt} & 0 & -\sin p \frac{dp}{dt} \end{pmatrix}; \quad \frac{dC}{dt} = \begin{pmatrix} -\sin (h) \frac{dh}{dt} & \cos (h) \frac{dh}{dt} & 0 \\ -\cos (h) \frac{dh}{dt} & -\sin (h) \frac{dh}{dt} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

An alternate equation can be derived to calculate the linear velocity due to rotational motion by noting that

$$V_{rot} = \frac{dR}{dt} = W \times R \tag{9}$$

where R and W are both expressed with respect to a fixed frame that is assumed to be (N,W,U). From previous explanations, it is known that

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_{\mathbf{x}} \\ \mathbf{R}_{\mathbf{y}} \end{pmatrix} = \mathbf{C}\mathbf{B}\mathbf{A}\mathbf{R}' \tag{10}$$

where R' is the same as the one that was already defined. W must be found, which is the rotation with respect to the fixed frame. Roll. pitch, and heading are given. Only heading, whose negative direction is up, can be considered in the fixed frame. The roll axis can be transformed into the fixed frame via rotations with magnitudes equal to pitch and heading, respectively, but in opposite directions. The pitch axis is parallel to the earth's horizontal axes and at an angle of the heading's magnitude clockwise from the West. Hence, the pitch axis can be transformed to the fixed frame by a rotation of magnitude equal to heading but in the opposite direction. Hence,

$$\mathbf{W} = \mathbf{CB} \begin{pmatrix} \frac{\mathbf{dr}}{\mathbf{dt}} \\ 0 \\ 0 \end{pmatrix} + \mathbf{C} \begin{pmatrix} 0 \\ \frac{\mathbf{dp}}{\mathbf{dt}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{\mathbf{dh}}{\mathbf{dt}} \end{pmatrix}$$
(11)

#### LINEAR VELOCITY DUE TO TRANSLATIONAL MOTION

In order to describe the linear velocity caused by surge, sway, and heave, a precise mathematical definition of these motions must be made. Since there seems to be some ambiguity on an accepted definition, they are defined in the next paragraph.

The surge, sway, and heave are oscillatory translational motions defined on a coordinate system whose origin is the mean location of the center of rotation that is assumed to be at or very near the mean location of the center of mass. The X-axis is the surge axis and points toward the direction of the mean velocity vector, if the vehicle is moving forward; and negative of this vector, if the vehicle is going backwards. The Z-axis is the heave axis and is pointed up with respect to the earth. The sway axis is horizontal and points in a direction that completes a right-handed system. As above, assume the vehicle mean velocity vector is constant. If the vehicle is not traveling, such as a ship floating in water, the X-axis is defined as the mean location of the centerline of the vehicle.

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Given the translational oscillations in the above coordinate system, these motions can be expressed in the (N,W,U) frame by the following transformation:

$$V_{\text{osc transl}} = \begin{pmatrix} V_{n} \\ V_{w} \\ V_{u} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_{x}' \\ V_{y}' \\ V_{z}' \end{pmatrix}$$
(12)

where  $\phi$  is the vehicle mean velocity angle from the North. As before, let positive  $\phi$  be the opposite of the convention, that is,  $\phi$  is defined as clockwise if  $\phi > 0$ .



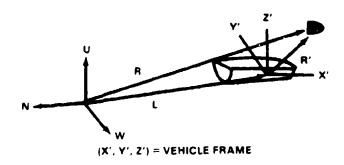


The remaining translational motion is the mean vehicle velocity itself, and it is not oscillatory. Given the mean speed and  $\phi$  as defined above, this velocity in the (N,W,U) frame is as follows:

$$V_{\text{mean vel}} = |V_{\text{mean vel}}| (\cos \phi, -\sin \phi, 0)$$
 (13)

#### **ACTUAL PROJECTILE VELOCITY**

The actual projectile velocity is the vector sum of the given velocity, which is the muzzle velocity  $V_{\text{muzzle}}$  plus all of the above linear velocities imparted to the projectile by vehicle motion at t=t, when the gun is fired. In the (N,W,U) frame



$$V_{\text{mussle}} = \left[ V_{\text{mussle}} \right] \left( \cos \left( \text{El}_{\text{o}} \right) \cos \left( A_{\text{g}}^{+} \right), - \cos \left( \text{El}_{\text{o}} \right) \sin \left( A_{\text{g}}^{+} \right), V \sin \left( \text{El}_{\text{o}} \right) \right)$$
 (14)

where

El = initial gun elevation

 $A_{*_0}^* = Az(initial) + Heading$ 

A, (initial) = gun bearing with respect to forward of the platform

Adding all the various velocities at  $t = t_o$ , the actual bullet velocity  $V_b$  is as follows:

$$V_{b} = V_{muzzle} + \left[V_{vehicle}\right]_{t=t_{o}}$$

$$= V_{muzzle} + \left[V_{rot} + V_{osc\ transl} + V_{mean\ vel}\right]_{t=t_{o}}$$
(15)

where b denotes bullet.  $V_{mean vel}$  is often assumed to be compensated after the first round, and so it may need to be removed in a computer model after the first round.

The correspondence of the terms of Equation (15) to that derived from Coriolis' of classical mechanics at  $t_0$  further demonstrates the truth of the former. Let R be the  $t_0$  bullet position in (N,W,U). Let R' be the  $t_0$  bullet position in the vehicle frame assuming for now R' = trunnion position at  $t_0$  and L be the vehicle frame's position in (N,W,U).

$$\frac{d\mathbf{R}}{dt} = \frac{d\mathbf{R}'}{dt} + \frac{d\mathbf{L}}{dt} \tag{16}$$

From Coriolis' equations in two forms,

$$\frac{d\mathbf{R}'}{dt} = \frac{d'\mathbf{R}'}{dt} + \mathbf{W} \times \mathbf{R}' \tag{17}$$

$$\frac{d\mathbf{R'}}{dt} = \mathbf{M} \frac{d'\mathbf{R'}}{dt} + \frac{d\mathbf{M}}{dt} \mathbf{R'}$$
(18)

where  $\frac{d}{dt}$  and  $\frac{d'}{dt}$  are derivatives in the (N,W,U) and (X', Y', Z') frames, respectively, M = rotation matrix from (X', Y', Z') to (N,W,U) and  $\mathbf{R}'_v$  is  $\mathbf{R}'$  in terms of (X', Y', Z'). Hence,

$$\frac{d\mathbf{R}}{dt} = \frac{d'\mathbf{R'}}{dt} + \mathbf{W} \times \mathbf{R'} + \frac{d\mathbf{L}}{dt} \text{ and also}$$
 (19)

$$\frac{dR}{dt} = M \frac{d'R'_v}{dt} + \frac{dM}{dt}R'_v + \frac{dL}{dt}$$
 (20)

Letting R' be in terms of (N,W,U),  $\frac{d'R'}{dt}$  of Equation (19) and M  $\frac{d'R'_v}{dt}$  of Equation (20) both equal  $V_{mussle}$  with respect to (X', Y', Z') at t but in terms of (N,W,U).

 $W \times R' = Wx$  (trunnion position) and

$$\frac{dM}{dt} R'_v = \frac{dM}{dt} \times (trunnion position)_v$$

both give  $V_{rot}$  at  $t_o$  with respect to and in terms of (N,W,U). Note that M is C B A and

$$\frac{dM}{dt} = CBA + CBA + CBA \text{ as before};$$

$$\frac{dL}{dt} = V_{mean vel} + V_{oscil trans} \text{ and}$$

$$\frac{d\mathbf{R}}{dt} = \mathbf{V}_{b}$$

Hence, Equations (19) and (20) contain exactly the terms on the right side of Equation (15) at  $t_a$ .

Since the gun is pivoted on the trunnion, the above holds even if  $R' \neq$  trunnion position at t<sub>o</sub>. In this case, one adds a fixed translation to (X', Y', Z') so that the new origin is the center of rotation of the bullet's initial position. The vector from this new location to the bullet position is still R' or  $R'_v$  and so all terms containing R' or  $R'_v$  are unchanged. Letting

 $\hat{L}$  = new L,  $\hat{L}$  = L + fixed translation implies that

$$\frac{\partial L}{\partial t} = \frac{\partial L}{\partial t}$$

Letting  $\hat{R} = \text{new } R$ ,  $\hat{R} = \hat{L} + R'$  since R = L + R', and so

$$\frac{d\hat{R}}{dt} = \frac{d\hat{L}}{dt} + \frac{dR'}{dt} = \frac{dL}{dt} + \frac{dR'}{dt} = \frac{dR}{dt}$$
(21)

Therefore, assuming  $R' \neq \text{trunnion position at } t_o \text{ changes nothing.}$ 

#### CALCULATION OF ERRORS

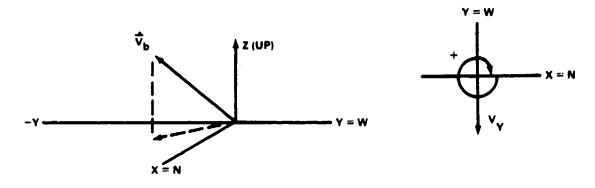
$$El_{new} = sin^{-1} \frac{V_{z_b}}{V_b}$$
 (22)

$$AZ_{new}^{*} = \cos^{-1} \frac{V_{x_{h}}}{\sqrt{V_{x_{b}}^{2} + V_{y_{b}}^{2}}} \quad \text{if } V_{y} < 0$$
 (23)

$$AZ_{\text{new}}^{*} = -\cos^{-1} \frac{V_{x_{b}}}{\sqrt{V_{x_{b}}^{2} + V_{y_{b}}^{2}}}$$
 (24)

if  $V_y > 0$ , since clockwise is defined positive in this application.

See the following diagram,



$$AZ_{new}^* = AZ_{new}^* + \text{Heading}$$
 (25)

The azimuth and elevation errors are as follows:

$$\epsilon_{AZ} = AZ^* - AZ_o^* \tag{26}$$

$$\epsilon_{E_1} = El_{RAW} - El_{C} \tag{27}$$

The distance errors for a flat trajectory are as follows:

$$e'_{AZ} = (RANGE) (\epsilon_{AZ})$$
 (28)

$$e'_{E_1} = (RANGE) (e_{E_1})$$
 (29)

Hence, we have modeled the gun shot errors caused by linear velocities imparted by vehicle motion. If one desires a more precise assumption than a flat trajectory, other formulas not given here can be used to find  $(e'_{AZ}, e'_{EI})$  when  $(e_{AZ}, e_{EI})$  as calculated above is given.

#### MODELS OF VEHICLE MOTION

A time series model of vehicle motion in water or land can be obtained experimentally. The following is a simple analytical model of vehicle motion in water, which is implemented in the model and may be useful in some applications.

$$r = roll = MAX_r \sin\left(\frac{2\pi}{T_r}t\right)$$
 (30)

$$p = pitch = MAX_p \sin\left(\frac{2\pi}{T_p} t\right)$$
 (31)

h = heading = MAX<sub>Y aw</sub> 
$$\sin\left(\frac{2\pi}{T_y} + t\right) + \phi$$
 (32)

where  $T_i$ s are periods and MAX<sub>i</sub>s are maximum amplitudes,  $\phi$  = angle of ship velocity vector (assumed constant), and t = time.

$$\dot{r} = \frac{2\pi}{T_r} \quad MAX_r \cos\left(\frac{2\pi}{T_r}t\right) \tag{33}$$

$$\dot{p} = \frac{2\pi}{T_p} \quad MAX_p \quad \cos\left(\frac{2\pi}{T_p} t\right) \tag{34}$$

$$\dot{h} = \frac{2\pi}{T_y} \quad MAX_{Yaw} \cos\left(\frac{2\pi}{T_y} t\right) \tag{35}$$

For surge, sway, and heave, only their derivatives are used, that is, their velocities. So,

$$V_{\text{surge}} = \frac{2\pi}{T_x} \text{ MAX (SURGE) } \cos\left(\frac{2\pi}{T_x} t\right)$$
 (36)

$$V_{sway} = \frac{2\pi}{T_y} \text{ MAX (SWAY) } \cos\left(\frac{2\pi}{T_y} t\right)$$
 (37)

$$V_{heave} = \frac{2\pi}{T_z}$$
 MAX (HEAVE)  $\cos\left(\frac{2\pi}{T_z}\right)$  (38)

where T<sub>i</sub>s are the periods and t = time.

#### IMPLEMENTATION OF THE MODEL

A computer program, AIMPT, was written in FORTRAN that gives the aimpoint error (azimuth and elevation) due to linear velocities, where the linear velocity imparted by rotational motion was modeled by the matrices method. Also, platform motion itself was simulated in a program called SHIP that uses a sinusoidal model. A program called HITPROB, written by the U.S. Army Materiel Systems Analysis Activity\* was used as a driver to the linear velocity model. This program simulates the probability of hitting a square or rectangular target by projectiles fired from a 25mm chain gun mounted on a Bradley Fighting Vehicle. Listings of AIMPT and SHIP are given below, and the program was run one time without linear velocities and one time with linear velocities. The listing of the program is in Appendix A.

<sup>&</sup>lt;sup>a</sup>Larry Bowman, A Methodology for Estimating Quasicombat Dispersions for Automatic Weapons. Interim Note G-103, U.S. Army Material Systems Analysis Activity, Aberdeen Proving Ground, Maryland, April 1982.

# Input parameters to AIMPT and SHIP were as follows:

Gun position	20° AZ 20° EI
Bullet muzzle velocity	1345 m/sec
Bearing at vehicle from north	60°
Maximum roll	0.0873 rad
Maximum pitch	0.0873 rad
Maximum yaw	0.0349 rad

Trunnion location from center of rotation (1., 1.73205, 1.73205) in m (meters)

Roll period	2 sec	Maximum sway amplitude	0.1 m
Pitch period	2 sec	Maximum heave amplitue	0.5 m
Yaw period	5 sec	Surge period	10 sec
Vehicle forward speed	4 m/sec	Sway period	10 sec
Maximum surge amplitude	0.1 m	Heave period	2 sec
		Time increment	0.6 sec

#### The output is as follows

Range	2000 m
Target size	2.286 x 2.286 m <sup>2</sup>
Hit probability without linear velocities	0.453
Hit probability with linear velocities	0.348

As seen, the probability of hit has been noticeably lowered by the presence of linear velocities.

#### **CONCLUSION**

The model explained in this report and implemented by the computer program described shows that linear velocities, imparted by gun platform motion on projectiles, influence the probability of hitting a target.

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#### RECOMMENDATION

In modeling gun fire accuracy, consideration should be given to the effects of linear velocities, imparted by platform motion, on accuracy. A model such as the one described in this report or actual test data can be used to model this effect.

#### BIBLIOGRAPHY

Bowman, Larry. Methodology for Estimating Quasicombat Dispersions for Automatic Weapons. Interim Note G-103, U.S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, Maryland, April 1982.

Britting, Kenneth R. Inertial Navigation Systems Analysis, 1971.

Encyclopedia of Science and Technology. McGraw-Hill, 1977.

Meyer, S. L. Effect of Roll and Pitch on Trajectory of Shells Fired From Stabilized Gun Tubes, TN/G-64/71, Naval Surface Weapons Center, Dahlgren, Virginia, 1971.

Naval Sea Systems Command. Standard Fire Control Symbols. OP 700 vol. I, Washington, D.C., 6 December 1957.

Symon, Keith R. Mechanics, 1960.

# APPENDIX A COMPUTER PROGRAM

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BUBROUTINE AIMPTINES

C WUEL DETERMINES WETHER TO ACC'T FOR SHIP VEL. NVEL=O MEANG VES

IMPLICIT REALAB (A-H. 0-2)

C AZOD, ELDO--:)NA AZ & ELEV RESP IN DEG, VEL-PPDJ SPEED, VELDEG-SHIP VEL

C ANOLE FRUM: NORTH, BDROLL, BDIT, BDVAL-MAX ROLL, PITCH, VAW RESP, TRUM-
C TRUMNION POS, TR. TP, TY-PERIOD OF ROLL, PITCH, VAW RESP, VSHIP-SHIP SPEED

COMMON 'SHIP1/TRUN(3), VELDEG, VEL, ELOD, AZOD, VAM, ROLL, PITCH, WROLL,
EMPITCH, WEAD, DVR(3), AZERR, ELERR, RANGE, MEADO, PI

COMMON 'SHIP2/YOUR, VSHIR, TSUR, 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            VELRAD-RADEACE-VELDEG
C HEAD-VELRAD-RADEACE-VAL
E-CAD-VERAD-VAL
C GET VEL, VECTOR OF BULLET: AZZ-MEADING AT TO PLUS AZ (FROM SHIP CTRLINE)
C AZOD. ELOO ARE EL AND AZ OF BULLET AT TIME TO
C HEAD-MEADING AT TIME TO
AZO-RADEACE-AZOD
ELO-RADEACE-CO
                                                                                                                                                                                                                                                                                                                                                           DIMENSION VI(3), V2(3), V3(3)
VELDEG=ANGLE OF SHIP VEL VECTOR FROM NORTH, VEL=VEL(MAC) OF BULLET,
DVR(3)=LINEAR VEL FROM ROLL, PITCH, AND YAM.
RADFAC=PI/180.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 VEL VECTOR FROM ROLL, PITCH, AND HEADING
DO 5 Jel. 3
DO 6 Jel. 3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             ADOT (2. 2) =-SIN(ROLL) ***ROLL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                AZZ=AZO+1EADO
VX=VEL+COB(ELO)+CDB(AZZ)
VY=-VEL+COB(ELO)+SIN(AZZ)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                8(1, 1) = COS(ROLL)

8(1, 1) = COS(PITCH)

8(1, 3) = SIN(PITCH)

8(3, 1) = -SIN(PITCH)

8(3, 3) = COS(PITCH)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    A(2, 2) =COS(ROLL)
A(2, 3) =SIN(ROLL)
A(3, 2) =-SIN(ROLL)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                C(2, 1) =- SIN(HEAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          B(2,2)=1 0
C(1,1)=C05(HEAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             C(1.2) =SIN(HEAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     C(2.2) =COS(HEAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  VZ=VEL+SIN(ELO)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DEGFAC=180, /P1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   BDOT(1, J)=0.
CDOT(1, J)=0.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ADOT (1, 3)=0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 A(1, 1)=1.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                C(3, 3) *1 0
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B(1, J) #0.
C(1, J) #0.
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C DVT IS THE LIN VEL FROM HEAVE, SURGE, IL SMAY, DVB-BHIP LIN VEL. CALC. NEW V OF PROJECTILE.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        CALL MATSUR(VI. VZ. TRANSI. 3.1)
C DVR 18, THE LINEAR WEL CAUSED BY ROLL. PITCH. & YAM
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              CALCULATE TRANSLATION VO. DVR. 3, 1)
C CALCULATE TRANSLATION VEL.
DVT(1) = VSUR+COS(VELRAD) + VSUA+SIN(VELRAD)
DVT(2) = - VSUR+SIN(VELRAD) + VSUA+COS(VELRAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               VXNEU=VX+DVR(1)+DVT(1)+DVB(1)
VXNEU=VX+DVR(2)+DVB(2)
VXNEU=VX+DVR(2)+DVB(2)
VXNEU=VX+DVR(2)+DVB(3)
C CALC. NEW EL AND AZ IN DEGREES.
VXYNEU=SQRT(VXNEU=P=2.+VYNEU=0.)
VMEU=SQRT(VXNEU=0.-.+VYNEU=0.)
                                                                                                                              BDDT(1, 3)=COB(PITCH) = MPITCH
BDDT(3, 1)=-COB(PITCH) = MPITCH
BDDT(1, 1)=-SIN(PEAD) = MPITCH
CDDT(1, 1)=-SIN(PEAD) = MPEAD
CDDT(1, 2)=-COS(NEAD) = MPEAD
CDDT(2, 1)=-COS(NEAD) = MPEAD
CDDT(2, 2)=-SIN(PEAD) = MPEAD
CALL MATPRO(8, TRANS1, 3, 3, 1)
CALL MATPRO(8, TRANS1, TRANS2, V1, 3, 3)
CALL MATPRO(CDDT, TRANS2, V1, 3, 2)
CALL MATPRO(CDDT, TRANS2, V1, 3, 2)
CALL MATPRO(CDT, TRANS1, 3, 3, 1)
CALL MATPRO(C, TRANS2, V2, 3, 3, 1)
CALL MATPRO(C, TRANS2, V3, 3, 3, 1)
CALL MATPRO(C, TRANS2, V3, 3, 3, 1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   WRITE(6. .) 'AZDIFF, ELDIFF', AZDIFF, ELDIFF
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            IF(RVEL EG.O)DV8(1)#VSHIP+COS(VELRAD)
IF(RVEL EG.O)DV8(2)#~VSHIP+SIN(VELRAD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ELNEW-DEGFAC+ELNEN
IF (VNEW, 0T. 0. )AZBTAR--AZBTAR
                             ADDT (3, 2) = -COS (NOLL) = HEOLL
ADDT (3, 3) = -SIN (NOLL) = HEOLL
BDDT (1, 1) = -SIN (PITCH) = HPITCH
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            AZDIFF =AZSTAR - (AZZ+DEOFAC)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                AZSTAR-ACOBIVXMEM/VXWEW)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Azerr-rande-radfacoazdiff
Elerr-rande-radfacoeldiff
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     AZSTAR-DEOFAC-AZSTAR
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VARIABLES

JOO1 SUBROUTINE SHIP(T) UOG2 IMPLICIT REAL+8 (A-H, D-Z) COMMON:SHIP1/TRUN(3), VELDEG, VEL, ELOD, AZOD, YAM, ROLL, PITCH, MPOLL,	COMMON SHIPZ/VBIR VBIR VBIR VBIR VBIR VBIR VBIR VBIR	ARGROLAS, PP. 17. 15UR. 15UR. 15UR. DI. BUGOM. BURER ARGROLAS, PP. 17R	ARCPIT+2, OPIVIDA ARCVAL+1) OPIVITA	OBTAINS SHIF ANOUAR POBITION	ROLL=BDROLL+SIN(ARGROL+T)	PITCH=8:PIT=SIN(ARQFIT=T)	YAL-BU-AL-BIN(AROYAL-I)	OBTAINS SHIP ANOULAR VELOCITY	WPDLL=1DROLL+ARGROL+COS(ARGROL+T)	₩PITCH -8DPIT*ARGPIT*COS(ARGPIT*)	LA MEAD=BDYALL AROYALL COS (AROYALL T)	OBTAINS SHIP TRANSLATIONAL VELOCITY	ARGSUR = 2 PP / TSUR	ANDRAM SO THE TANK TH	ARCHEA*; *P1/THEA	VSUR=813UR+6CBC ( ARGBUR+1)	VSMA=B⊡:NA=ARGSMA=COS ( ARGSMA=T )	VHEA=BDHEA=6ABGHEA=CDS(ARGHEA=T)	RETURN		
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PROGRAM SECTIONS

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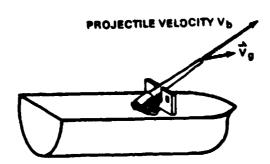
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## APPENDIX B

# VELOCITY IMPARTED ON A PROJECTILE AS IT TRAVELS THROUGH A GUN TUBE

A source of additional velocity not yet analyzed in this report is the velocity imparted by angular motion as the projectile travels through the gun tube. This velocity,  $V_{gun}$ , is caused by the vehicle angular velocity continuing to impart linear velocity when all or part of the moving bullet is still in the tube.



Since the component of  $V_{gun}$  that is parallel to  $V_b$  does not totally add onto the projectile, only the component of  $V_{gun}$  normal to  $V_b$  completely adds to  $V_b$ . Call this component  $V_g$ . Hence  $|V_{gun}| = \max |V_g|$ . Since the stabilized gun tube does not rotate with the vehicle, the magnitude of the radius of rotation on the tube can be represented by  $V_{rot}$  defined previously in this report. Hence, given that  $t_2 - t_1$  represents the elapsed time from discharge to the time the bullet completely exits the gun, we have the following equation

$$V_{gun} = \int_{t_1}^{t_2} \dot{V}_{rot}(t)dt = V_{rot}(t_2) - V_{rot}(t_1)$$
 (A-1)

This velocity is generally very small compared to  $V_{rot}$ , since  $t_2 - t_1$  is small and so  $V_{rot}$  ( $t_2$ )  $\approx V_{rot}$  ( $t_1$ ). Hence, neither  $V_g$  nor  $V_{gun}$  is entered in the model.

## APPENDIX C

# FORMULA FOR DIFFERENTIATING A PRODUCT OF MATRICES TIMES A VECTOR

Theorem: 
$$\frac{d}{dt} \prod_{i=1}^{n} A_{i} R = \sum_{i=1}^{n} (A_{i} ... \frac{dA_{i}}{dt} ... A_{n} R) + \prod_{i=1}^{n} A_{i} \frac{dR}{dt}$$
 (B-1)

Proof:

Mathematical induction is used.

(case for 
$$n = 1$$
) Let  $AR = (a_{ij})$   $(R_j) = \begin{pmatrix} \sum_{j=1}^{n} a_{ij} & R_j \\ \vdots & \vdots \\ \sum_{j=1}^{n} a_{nj} & R_j \end{pmatrix}$  (B-2)

$$\frac{dAR}{dt} = \begin{bmatrix}
n \\ \sum_{j=1}^{n} \left(\frac{da_{ij}}{dt} R_{j} + a_{ij} \frac{dR_{j}}{dt}\right) \\
\vdots \\
n \\ \sum_{j=1}^{n} \frac{da_{nj}}{dt} R_{j} + a_{nj} \frac{dR_{i}}{dt}
\end{bmatrix} = \Gamma$$

$$\left(\frac{dA}{dt}\right) R = \left(\frac{da_{ij}}{dt}\right) \left(R_{j}\right) = \begin{pmatrix}
n \\ \sum_{j=1}^{n} \frac{da_{ij}}{dt} R_{j} \\
\vdots \\
\sum_{j=1}^{n} \frac{da_{nj}}{dt} R_{j}
\end{pmatrix}$$
(B-4)

$$\left(\frac{dA}{dt}\right) R = \left(\frac{da_{ij}}{dt}\right) \left(R_{j}\right) = \begin{pmatrix} \sum_{j=1}^{n} \frac{da_{ij}}{dt} R_{j} \\ \vdots \\ \sum_{j=1}^{n} \frac{da_{nj}}{dt} R_{j} \end{pmatrix}$$
(B-4)

$$A_{\overline{dt}}^{\overline{dR}} = \begin{pmatrix} n & dR_{ij} \\ \sum_{j=1}^{n} a_{ij} & dR_{ij} \\ \vdots & \vdots & \vdots \\ \sum_{n_{j}} dR_{j} & dt \end{pmatrix}$$
(B-5)

$$\therefore \frac{dA}{dt} R + A \frac{dR}{dt} = \begin{bmatrix} \frac{n}{\sum_{j=1}^{n} \left( \frac{da_{ij}}{dt} R_j + a_{ij} \frac{dR_j}{dt} \right)}{\vdots} \\ \vdots \\ \frac{n}{j=1} \frac{da_n}{dt} R_j + a_{nj} \frac{dR_j}{dt} \end{bmatrix} = \Gamma$$
(B-6)

$$\therefore \frac{AR}{dt} = \frac{dA}{dt} R + A \frac{dR}{dt}$$
 (B-7)

(case for n+1 matrices if true for n matrices)

Suppose

$$\frac{d}{dt} \prod_{i=1}^{n} A_i R = \sum_{i=1}^{n} \left( A_i \dots \frac{dA_i}{dt} \dots A_n R \right) + \prod_{i=1}^{n} A_i \frac{dR}{dt}$$
(B-8)

$$\frac{d}{dt} \prod_{i=1}^{n+1} A_i = \frac{dA_i}{dt} \begin{pmatrix} n+1 \\ \Pi \\ i=2 \end{pmatrix} + A_i \frac{d}{dt} \begin{pmatrix} n+1 \\ \Pi \\ i=2 \end{pmatrix}$$

$$(B-9)$$

Without loss of generality, the subscripts can be renamed as  $ke \{0, ..., n\}$  and then back to  $ie \{1, ..., n+1\}$ 

$$\frac{d}{dt} \prod_{i=1}^{n+1} A_{i} R = \frac{d}{dt} A_{o} \prod_{k=1}^{n} A_{k} R = \frac{dA_{o}}{dt} \left( \prod_{k=1}^{n} A_{k} R \right) + A_{o} \frac{d}{dt} \prod_{k=1}^{n} A_{k} R \qquad (B-10)$$

$$\approx \frac{dA_{o}}{dt} \left( \prod_{k=1}^{n} A_{k} R \right) + A_{o} \left[ \prod_{k=1}^{n} A_{k} \frac{dR}{dt} + \sum_{k=1}^{n} \left( A_{1} \dots \frac{dA_{k}}{dt} \dots A_{n} R \right) \right]$$

$$\approx \frac{dA_{o}}{dt} \left( \prod_{k=1}^{n} A_{k} R \right) + A_{o} \left[ \prod_{k=1}^{n} A_{k} \frac{dR}{dt} \right] + \sum_{k=1}^{n} \left( A_{o} A_{1} \dots \frac{dA_{k}}{dt} \dots A_{n} R \right)$$

$$\approx \frac{dA_{1}}{dt} \prod_{i=2}^{n+1} A_{i} R + \left( \prod_{i=1}^{n+1} A_{i} \frac{dR}{dt} \right) + \sum_{i=2}^{n+1} \left( A_{1} A_{2} \dots \frac{dA_{i}}{dt} \dots A_{n+1} R \right)$$

$$= \prod_{i=1}^{n+1} A_i \frac{dR}{dt} + \frac{dA_1}{dt} \prod_{i=2}^{n+1} A_i R + \sum_{i=2}^{n+1} \left( A_1 A_2 \dots \frac{dA_i}{dt} \dots A_n R \right)$$

$$= \prod_{i=1}^{n+1} A_i \frac{dR}{dt} + \sum_{i=1}^{n+1} \left( A_1 \dots \frac{dA_i}{dt} \dots A_{n+1} R \right) \Rightarrow \text{ true for } n+1 \text{ matrices.}$$

Thus, by mathematical induction, the theorem is true.

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